Functional Discrimination by Wavelets

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Abstract

Classification or discrimination is an important statistical problem in various fields of natural and social sciences. Several techniques have been proposed for analyzing multivariate observations such as Fisher’s linear discriminant analysis (LDA) and quadratic discriminant analysis (QDA). It is often the case that the dimension of the covariates is quite large, while the whole population of the training data set is relatively small. In such cases, the methods of multivariate analysis are not directly applicable because the variance-covariance matrix becomes singular, and the Mahalanobis distance cannot be calculated.

In this paper, we introduce a functional discriminant approach. Functional data analysis was proposed by Ramsay and Silverman (1997), and has been applied in various fields such as biomechanics, chemometrics, meteorology, and so on. Basis expansion approaches, such as Fourier and spline bases, have been very popular in this field, while more recently; radial basis expansions have also been considered by Araki et al. (2004). Here, however, we believe that the local adaptivity of wavelet-based curve estimation may yield favorable results when the curves have irregular and complex structures. Wavelets form an orthonormal basis and enable multiresolution analyses by localizing a function in different phases of both time and frequency domains simultaneously and thus offer some advantages over traditional Fourier expansions. Theoretical and practical developments of their use in statistics have been made by Donoho et al. (1995, 1996), Hall and Patil (1996), Johnston and Silverman (1997) among others.

We use a wavelet-based smoothing technique to obtain a set of functional data from discretely sampled observations of different individuals, and then we consider the logistic discriminant analysis which is previously introduced for functional data by Araki et al. (2004). Estimation of the model is based on a regularized log-likelihood method, where we apply the model selection criteria derived for the wavelet-based functional logistic model. This procedure is illustrated in a numerical example given by an application to digitized analog signals of “phonemes”, where this problem forms the subject of sound recognition in signal analysis.

Key words: functional data analysis, logistic discrimination, regularization, wavelet-based smoothing technique.

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