

# The Generalized $t$ -Distribution on the Circle

Hai-Yen Siew<sup>1,\*,\dagger</sup>, Shogo Kato<sup>2,\dagger</sup> and Kunio Shimizu<sup>2</sup>

<sup>1</sup> Department of Statistical Science, The Graduate University for Advanced Studies,  
4–6–7 Minami Azabu, Minato-ku, Tokyo 106–8569, Japan.

<sup>2</sup> Department of Mathematics, Keio University,  
3–14–1 Hiyoshi, Kohoku-ku, Yokohama 223–8522, Japan.

## Abstract

An extended version of  $t$ -distribution on the unit circle is generated by conditioning a normal mixture distribution, which is broadened to include not only unimodality and symmetry, but also bimodality and asymmetry, depending on the values of parameters. After reparametrization, the distribution contains four circular distributions as special cases: symmetric Jones–Pewsey, generalized von Mises, generalized cardioid and generalized wrapped Cauchy distributions. As an illustrative example, the proposed model is fitted to the number of occurrences of the thunder in a day.

**Key words:** Cardioid distribution; Directional statistics; Generalized von Mises distribution; Normal mixture distribution; Wrapped Cauchy distribution

\*Corresponding author

E-mail address: haiyen@ism.ac.jp (Hai-Yen Siew)

## 1. Introduction

Circular data generally refer to the observations which can be expressed as points on the circle. Typical examples are wind directions and locations of earthquake. Additionally, the periodic data, such as hours of a day and days of a year, are also viewed as the circular data after being converted into angles. Among some well-known circular distributions (Mardia and Jupp, 1999; Jammalamadaka and SenGupta, 2001), the von Mises distribution plays an important role as a circular normal distribution for possessing some analogous properties as the normal distribution on the real line. It is unimodal and symmetric about its mean direction. Since the circular data are not always unimodal and symmetric, the distribution is then extended to a more general form which covers not only unimodality and symmetry, but also bimodality and asymmetry. This extension is called the generalized von Mises distribution (Maksimov, 1967; Rukhin, 1972) and has been studied in detail by Yfantis and Borgman

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<sup>\dagger</sup> Now at The Institute of Statistical Mathematics,  
4–6–7 Minami Azabu, Minato-ku, Tokyo 106–8569, Japan.